|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **ECE 3300 EXAM 2 NOTES SHEET Revision 23** | | | | | | | | | | | | | |
| The output of a system due to the input is called the **impulse response** and is denoted *h(t)*. (Same for discrete) | The output of a system due to the inputis called the **step response** and is denoted *g(t).* | | | | | | Input    Output *y(t)* | | | | **Impulse and Step Response** | | |
| Impulse and/or step response do not uniquely define a system.  **Memoryless System**  For all inputsand for all times *t* or *n*, the output *y(t)*/*y[n]* depends on the input /*x[m]*  at no times other than /*m=n*. |  | | | | | | **MEMORY**  Memoryless Systems  >No dependence on past or future  >No storage inputs required  Systems with memory  >At least one possible output depends on the future or past |  | | | | |
| In the system given by:  It is memoryless because *y(t)* always depends on *x(t)* at time t ONLY.  The *t+1* inside cos(t+1) is irrelevant | In the system given by:  The system is not memoryless because y[3]=x[2]  Continuous time in red  Discrete time in blue | | | | **Causal Systems**  For all inputs times t or n, the output y(t)/x[m] depends on the input /y[n] at times for which  Only what is **inside** matters. | | | Causal systems have no dependence on the future while *noncausal* systems have at least one output that depends on the future. | | | |
|  | | Stable systems (AKA bounded-input bounded-output (BIBO) stable)   * Guarantee that a finite voltage or current at the input will not lead to an infinite voltage or currrent at the output * This can be proven by determining *B* as a function of *A* | | | | A non-real-time system can be noncausal. An example would be a DVD player because they look “ahead” of what is being outputted in order to interpolate and collect corrupted data | | Determine whether the system given by  *y(t) = 2x(-t­­2) is causal.* | | | |
| **Example of memoryless, causality, and stability**  LTI w/ impulse response   * System is *memoryless* only if . This system has other terms so it is **not memoryless** * System is *causal* if for all . This is true for this system, since the unit step is only on for , so the system is **causal**. * System is *stable* onlif if the area of is finite.   Since the answer is infinite, it is **not stable**. | | | | | | Unstable systems:  -Can be severely damaged by bounded inputs  -Proven by finding a single example of a bounded input that produces an unbounded output | | | |  | |
| Determine whether the system given by is stable. | | | |
| Invertible systems do not destroy any information in any input signals and the are proven by determining the inverse system.  Non-invertible systems destroy some information in some input signals and they are proven by determining two different inputs that result in the same output. | | | | | | Determine whether the system given by is invertible.    System is thus not invertible | | | | Determine whether the system given by *y[n] = (n+1)x[n]*  is invertible. | |
| **Time-Invariant Systems**  For all inputs and all times t0 and t/n0 and n, the input produces thet output at time t/n  - For time-invariant systems, delaying a given input has no effect on the output except a delay of the same amount  - For time-varying systems, delaying at least one input by at least one value changes the output in some way other than a delay of the same amount | | | | | | Proving Time-(In)Variant System   1. Starting with original system, replace every with 2. Starting with original system, replace every t/n with t-t0*/*n-n0 3. If identical, system is *time invariant*   To prove a system in time-varying, show that the above rules fail or give a single example of an input that when delayed by a single amount, does not only delay the output by that amount. | | | | Determine whether the system given by  is time-invariant.    Determine if system is time invariant for | |
| **Linear Systems**   1. Starting with original system, replace every with 2. Starting with original system, determine 3. If identical, system is linear.   Show the above **fails** or give a single example of failure.  **Proving a system in *nonlinear***   1. Doubling the input should double the output (a1=2, a2=0) 2. An input of zero should give an output of zero (a1=0, a2=0) | | | | Determine whether the system y(t)=x(t+1)-1 is linear.   1. Replace with : 2. Determine   =   1. Check to see if the two equations (orange) are the same. They aren’t, so the system is *nonlinear.*   Unrelated: produces a reverse unit impulse at | | | | | | | Calculationg Continuous-Time Convolutions   1. Determine and and enter into integral. 2. Determine effects of step functions that depend only on . 3. Determine effects of step functions that depend on and . 4. Determine regions of integration as functions of  *t* and integrate for each case.   Calculating Discrete-Time Convolutions   1. Determine x[m] and h[n-m] and enter into sum. 2. Determine effects of step functions that depend only on m. 3. Determine effects of step functions that depend on n and m. 4. Determine regions of summation as functions of n and sum for each case. |
| Linear Time-Invariant (LTI) Systems  Convolution Integral + Shorthand Notation    Convolution Sum + Shorthand Notation  LTI systems are **completley** characterized by their impulse response. To know *h* is to know the system. | | | | Example Discrete Time Convolution | | | | | | |  |
| Checking continuous-time convolutions  **Check #1**  If x(t) and h(t) have no , y(t) must have no jumps  **Check #2**  If x(t) is on over (a,b) and h(t) is on over (c,d) then can only be on over (a+c, b+d)  Checking Discrete-Time Convolution  **Check #1**  If there are choices in setting region boundires, all choices should give the same answer  **Check #2**  If x[n] is on over [a,b] and h[n] is on over [c,d] then (x can only be on over [a+c, b+d] | | |  | | | | | | | |  |
| Example Continuous Time Convolution | | | Convolution with a Unit Impulse | | | | | | **Various MCS Problems**   1. The system is **stable** because a bounded input guarantees a bounded output. 2. The system is **neither** causal or memoryless. *Look at both positive and negative time.* 3. The system is **causal** and **stable**, but **not memoryless**. 4. A system with input and output . This system is **stable** 5. A system with input and output . It is **neither memoryless** **nor** **causal**. 6. is **neither memoryless nor causal**. Reason: | | |
| Example using properties to find convolutions | | | **Convolutions Involving Impulses**  *Applies for discrete time as well.*  **System Properties Example**   * System **not memoryless** since * System **not causal** for same reason * System **not stable** since: | | | | | | Convolution Properties  Superposition  Amplitude Scaling  Time Delay Property  Difference/Differentiation Property  and  Integration/Summation Property  and | | |
| **Output of LTI Systems with Periodic Inputs**    If system 1 is in **series** with system 2, use **convolution**.  If system 1 is in **parallel** with system 2, **add** the 2 systems. | | |  | | | | | | | |  |
| **NOTE: Periodic LTI Systems**  For a periodic input with period , the range of the fundamental cycle, , would be or equivalently | | | **Linear Systems Definition**  For all inputs and , all constants and , and all times , the input produces the output | | | | | | | | **Stability Example**  The system is **unstable** because the input for all gives an unbounded output |